

Exercice 2 :

Soient $z = 4 - 2i$ et $w = 2 + i$

Déterminez

$$z_1 = z + 3w$$

$$z_2 = w + \bar{w}$$

$$z_3 = 3z - w$$

$$z_5 = z \bar{w}$$

$$z_6 = z \bar{z}$$

$$z_4 = \bar{z} - z$$

$$z_7 = \frac{z - i}{1 + w}$$

Que remarquez-vous ?

Généralisez votre remarque.

Exercice 2 : $z = 4 - 2i$ et $w = 2 + i$

$$z_1 = z + 3w = 4 - 2i + 3(2 + i)$$

$$= 4 - 2i + 6 + 3i = \mathbf{10 + i}$$

$$z_2 = w + \bar{w} = (2 + i) + (2 - i) = \mathbf{4}$$

$$z_3 = 3z - w = 3(4 - 2i) - (2 + i)$$

$$= 12 - 6i - 2 - i = \mathbf{10 - 7i}$$

$$z_4 = \bar{z} - z = (4 + 2i) - (4 - 2i)$$

$$= 4 + 2i - 4 + 2i = \mathbf{4i}$$

$$z_5 = z \bar{w} = (4 - 2i)(2 - i) = 4(2 - i) - 2i(2 - i)$$

$$= 8 - 4i - 4i + 2i^2 = 8 - 8i + 2(-1) = \mathbf{6 - 8i}$$

Exercice 2 : $z = 4 - 2i$ et $w = 2 + i$

$$z_6 = z \bar{z} = (4 + 2i)(4 - 2i) = 4^2 - (2i)^2 \\ = 16 - 4i^2 = 16 - 4(-1) = \mathbf{20}$$

$$z_7 = \frac{z - i}{1 + w} = \frac{(4 - 2i) - i}{1 + (2 + i)} = \frac{4 - 3i}{3 + i}$$

$$= \frac{(4 - 3i)(3 - i)}{(3 + i)(3 - i)} = \frac{12 - 9i - 4i + 3i^2}{3^2 - i^2}$$

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Que remarquez-vous ?

Démontrez les conjectures.

$$z_1 = z + 3w = 10 + i$$

$$z_2 = w + \bar{w} = 4$$

$$z_3 = 3z - w = 14 - 5i$$

$$z_4 = \bar{z} - z = 4i$$

$$z_5 = z \bar{w} = 6 - 8i$$

$$z_6 = z \bar{z} = 20$$

$$z - i$$

$$z_7 = \frac{z - i}{1 + w} = 0,9 - 1,3i$$

Que remarquez-vous ?

Démontrez les conjectures.

$$z_1 = z + 3w = 10 + i$$

$$z_2 = w + \bar{w} = 4 \text{ réel}$$

$$z_3 = 3z - w = 14 - 5i$$

$$z_4 = \bar{z} - z = 4i \text{ imaginaire pur}$$

$$z_5 = z \bar{w} = 6 - 8i$$

$$z_6 = z \bar{z} = 20 \text{ réel}$$

$$z_7 = \frac{z - i}{1 + w} = 0,9 - 1,3i$$

Que remarquez-vous ?

Démontrez les conjectures.

$$\begin{aligned}w + \bar{w} &= (a + bi) + (a - bi) \\&= a + bi + a - bi = \mathbf{2a} \quad \text{réel}\end{aligned}$$

$$\begin{aligned}z - \bar{z} &= (a + bi) - (a - bi) \\&= a + bi - a + bi = \mathbf{2bi} \quad \text{imaginaire pur}\end{aligned}$$

$$\begin{aligned}z \bar{z} &= (a + bi)(a - bi) = a^2 - (bi)^2 \\&= a^2 - b^2 i^2 = a - b^2 (-1) = \mathbf{a^2 + b^2} \\&\quad \text{réel}\end{aligned}$$

Que remarquez-vous ?

Démontrez les conjectures.

$$\begin{aligned}w + \bar{w} &= (a + bi) + (a - bi) \\&= a + bi + a - bi = \mathbf{2a} \text{ réel}\end{aligned}$$

$$\begin{aligned}z - \bar{z} &= (a + bi) - (a - bi) \\&= a + bi - a + bi = \mathbf{2bi} \text{ imaginaire pur} \\&\quad (\text{ si } b \neq 0)\end{aligned}$$

$$\begin{aligned}z \bar{z} &= (a + bi) (a - bi) = a^2 - (bi)^2 \\&= a^2 - b^2 i^2 = a - b^2 (-1) = \mathbf{a^2 + b^2} \\&\quad \text{réel positif}\end{aligned}$$