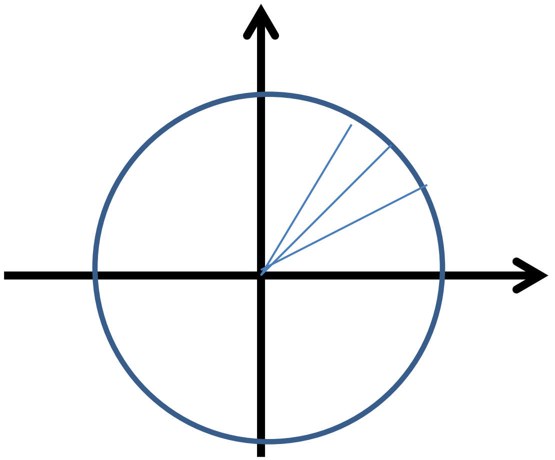
Exercice 11 :

2°) Complétez les égalités :

$$\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x = \cos(x + \ldots \quad)$$

$$\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x = \sin(x + \ldots \quad)$$

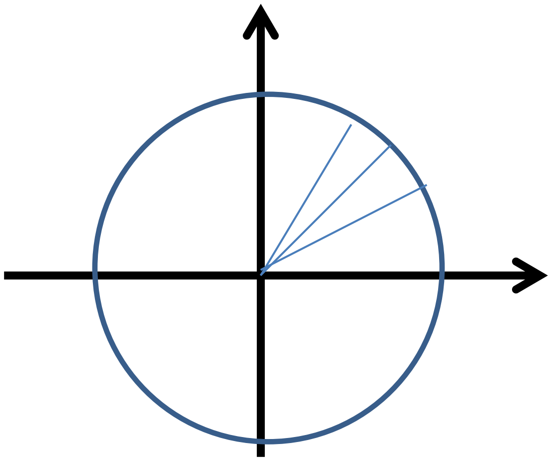
$$\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x = \cos(x + \ldots)$$



$$\ldots = \cos(b + a)$$

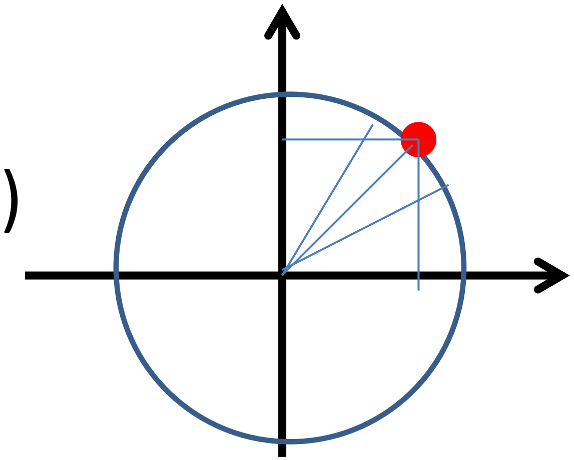
$$\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x = \cos(x + \ldots)$$

$$\cos a \, \cos b - \sin a \, \sin b = \cos(b + a)$$



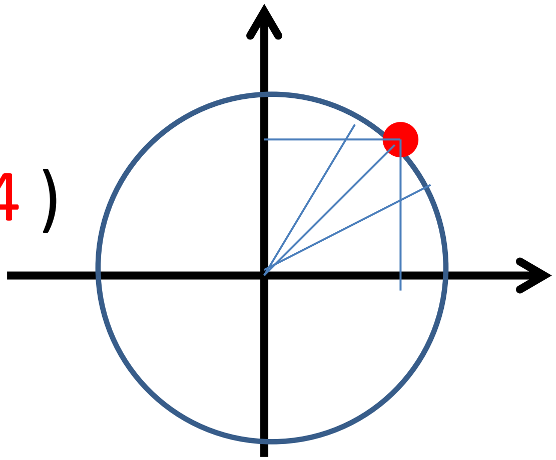
$$\boxed{\frac{\sqrt{2}}{2}} \cos x - \boxed{\frac{\sqrt{2}}{2}} \sin x = \cos(x + \ldots)$$

$$\boxed{\cos a} \cos b - \boxed{\sin a} \sin b = \cos(b + a)$$



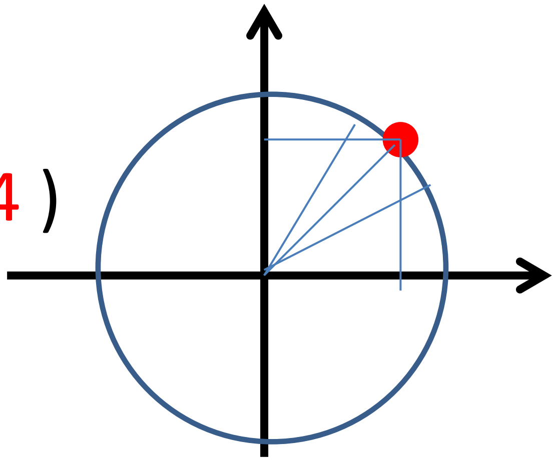
$$\boxed{\frac{\sqrt{2}}{2}}\cos x - \boxed{\frac{\sqrt{2}}{2}}\sin x = \cos\left(x + \pi/4\right)$$

$$\boxed{\cos a}\cos b - \boxed{\sin a}\sin b = \cos(b + a)$$



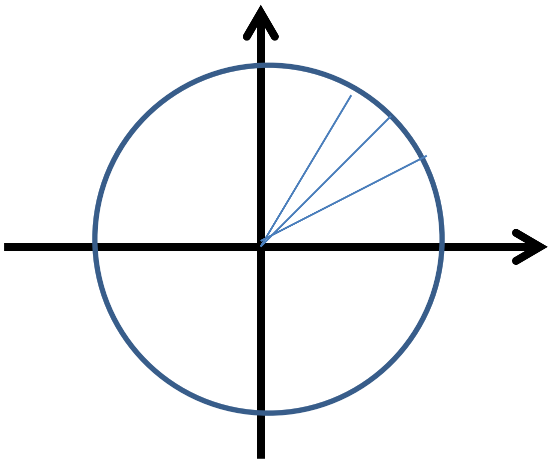
$$\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x = \cos(x + \pi/4)$$

$$\cos a \cos b - \sin a \sin b = \cos(b + a)$$



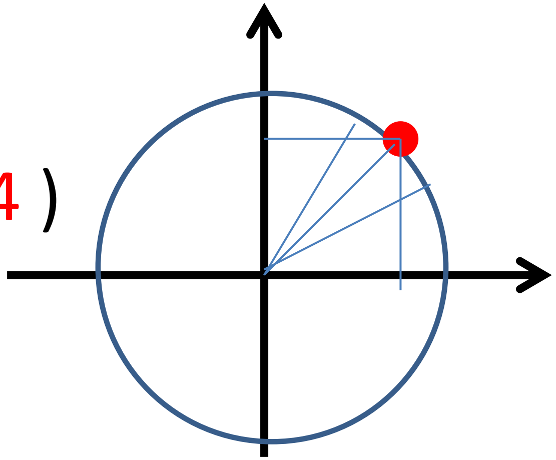
$$\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x = \sin(x + \ldots)$$

$$\ldots = \sin(b + a)$$



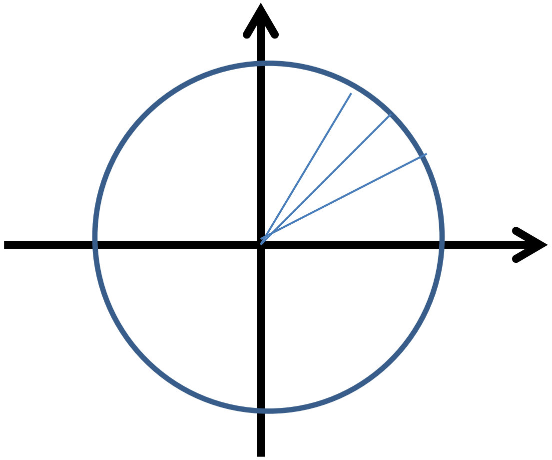
$$\frac{\sqrt{2}}{2} \cos x - \frac{\sqrt{2}}{2} \sin x = \cos(x + \pi/4)$$

$$\cos a \, \cos b - \sin a \, \sin b = \cos(b + a)$$

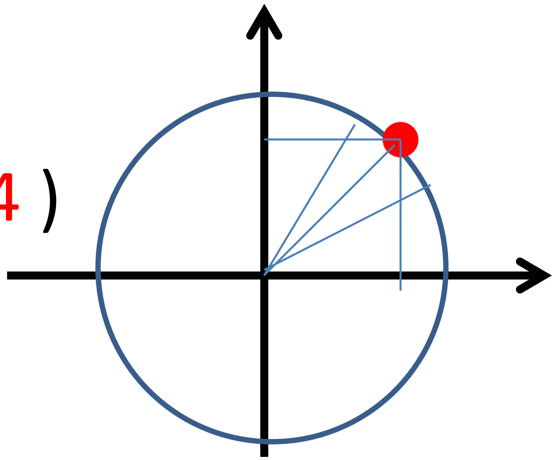


$$\frac{\sqrt{2}}{2} \cos x - \frac{\sqrt{2}}{2} \sin x = \sin(x + \ldots)$$

$$\sin a \, \cos b + \cos a \, \sin b = \sin(b + a)$$

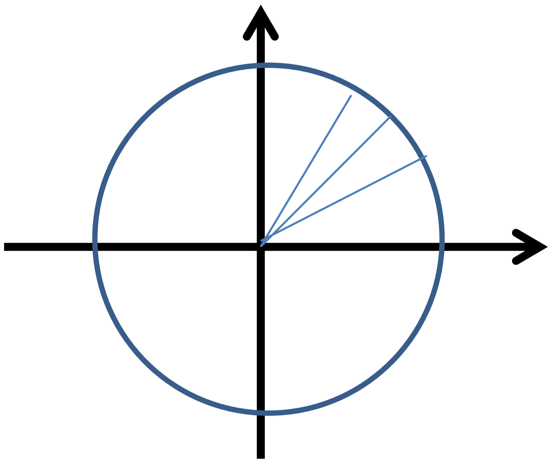


$$\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x = \cos(x + \pi/4)$$



$$\cos a \cos b - \sin a \sin b = \cos(b + a)$$

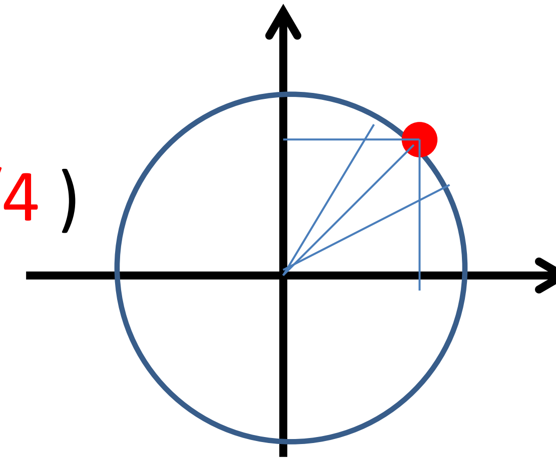
$$\frac{\sqrt{2}}{2}\cos x + \frac{-\sqrt{2}}{2}\sin x = \sin(x + \ldots)$$



$$\sin a \cos b + \cos a \sin b = \sin(b + a)$$

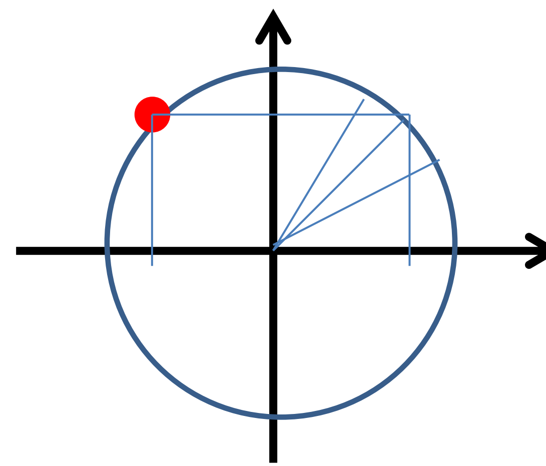
$$\frac{\sqrt{2}}{2} \cos x - \frac{\sqrt{2}}{2} \sin x = \cos(x + \pi/4)$$

$$\cos a \; \cos b - \sin a \; \sin b = \cos(b + a)$$

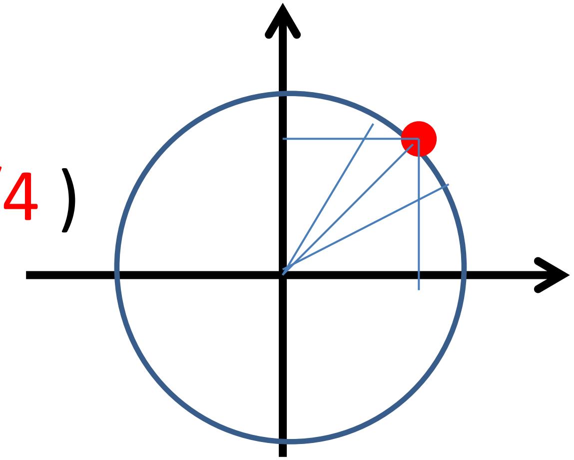


$$\boxed{\frac{\sqrt{2}}{2}} \cos x + \boxed{\frac{-\sqrt{2}}{2}} \sin x = \sin(x + \ldots)$$

$$\boxed{\sin a} \; \cos b + \boxed{\cos a} \; \sin b = \sin(b + a)$$

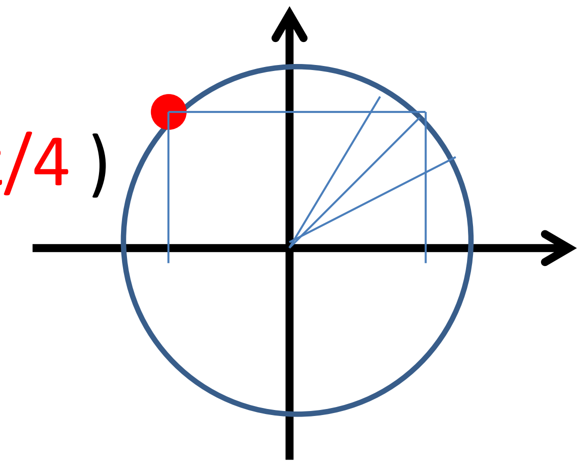


$$\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x = \cos(x + \pi/4)$$



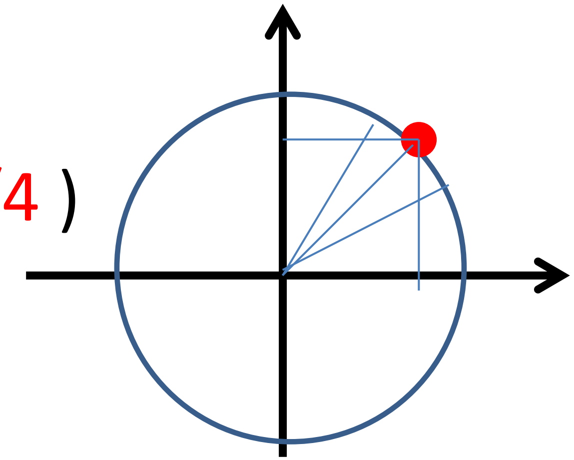
$$\cos a \, \cos b - \sin a \, \sin b = \cos(b + a)$$

$$\boxed{\frac{\sqrt{2}}{2}}\cos x + \boxed{\frac{-\sqrt{2}}{2}}\sin x = \sin(x + 3\pi/4)$$



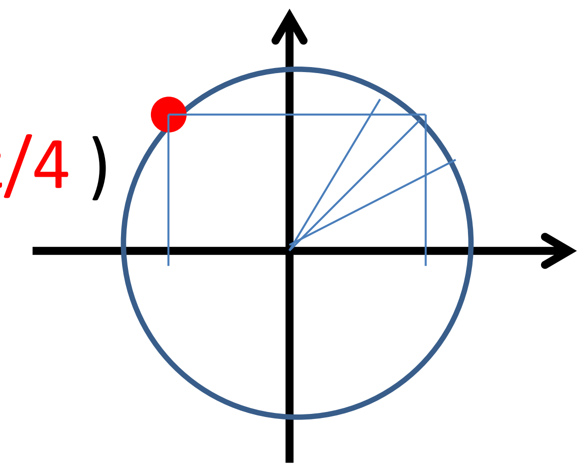
$$\boxed{\sin a}\cos b + \boxed{\cos a}\sin b = \sin(b + a)$$

$$\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x = \cos(x + \pi/4)$$



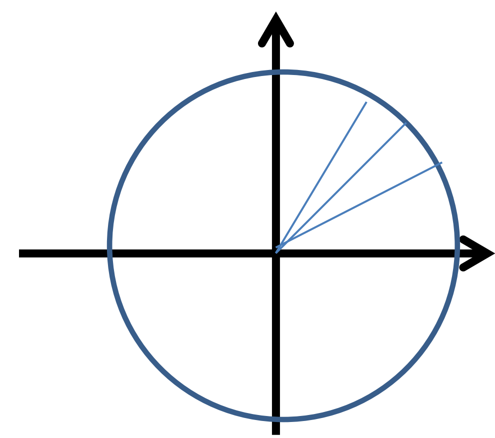
$$\cos a \, \cos b - \sin a \, \sin b = \cos(b + a)$$

$$\frac{\sqrt{2}}{2}\cos x + \frac{-\sqrt{2}}{2}\sin x = \sin(x + 3\pi/4)$$

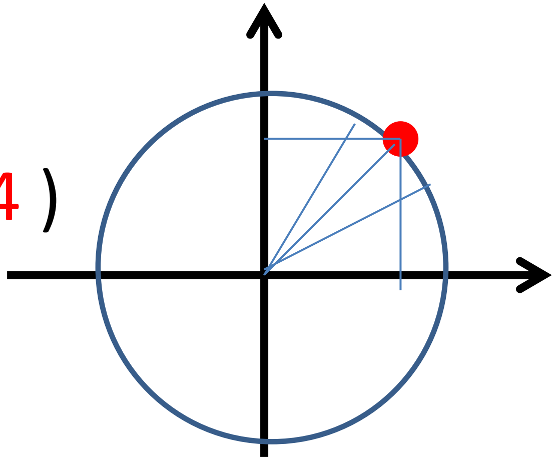


$$\sin a \, \cos b + \cos a \, \sin b = \sin(b + a)$$

$$\frac{-\sqrt{2}}{2}\cos x + \dots \sin x = \sin(x - \pi/4)$$

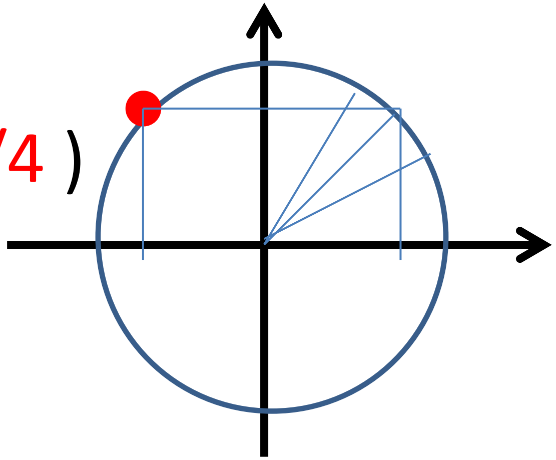


$$\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x = \cos(x + \pi/4)$$



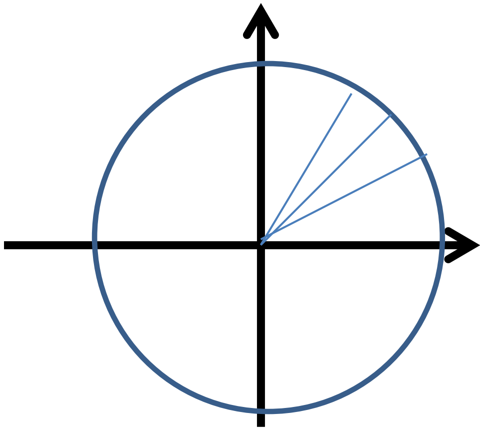
$$\cos a \cos b - \sin a \sin b = \cos(b + a)$$

$$\frac{\sqrt{2}}{2}\cos x + \frac{-\sqrt{2}}{2}\sin x = \sin(x + 3\pi/4)$$



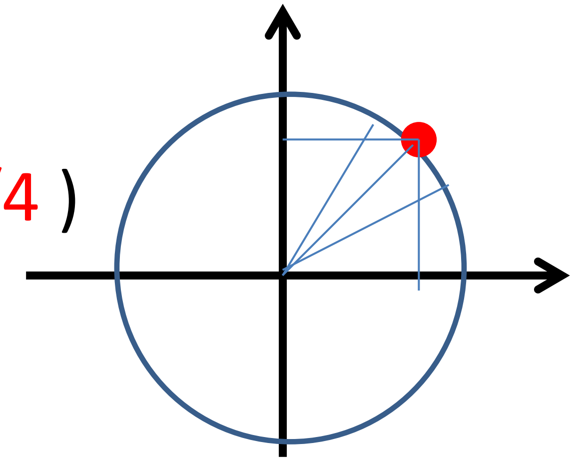
$$\sin a \cos b + \cos a \sin b = \sin(b + a)$$

$$\frac{-\sqrt{2}}{2}\cos x + \ldots \quad \sin x = \sin(x + (-\pi/4))$$



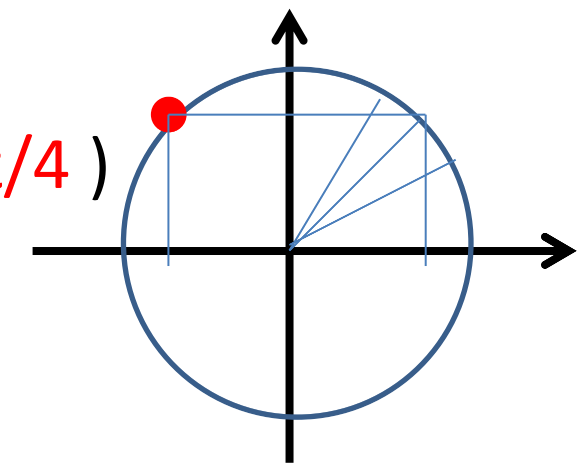
$$\frac{\sqrt{2}}{2} \cos x - \frac{\sqrt{2}}{2} \sin x = \cos (x + \pi/4)$$

$$\cos a \ \cos b - \sin a \ \sin b = \cos(b + a)$$



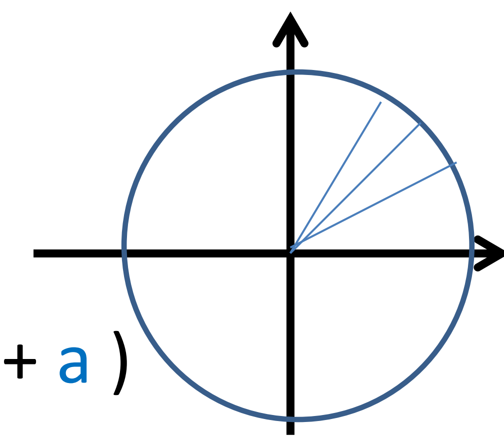
$$\frac{\sqrt{2}}{2} \cos x + \frac{- \sqrt{2}}{2} \sin x = \sin (x + 3\pi/4)$$

$$\sin a \ \cos b + \cos a \ \sin b = \sin(b + a)$$



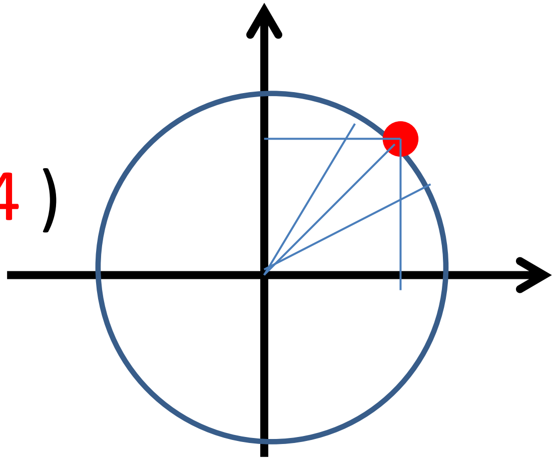
$$\frac{- \sqrt{2}}{2} \cos x + \ldots \quad \sin x = \sin (x + (- \pi/4))$$

$$\sin (- \pi/4) \cos b + \cos (- \pi/4) \sin b = \sin(b + a)$$



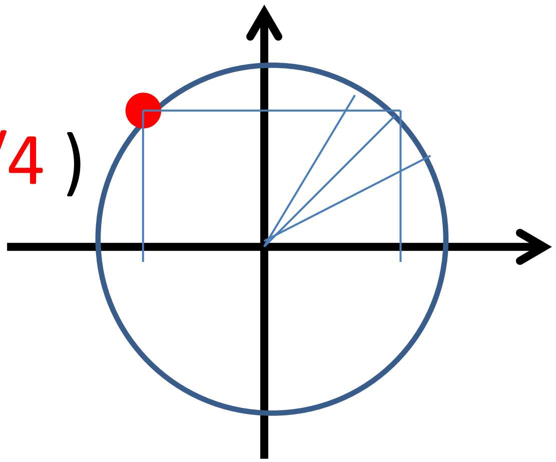
$$\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x = \cos\left(x + \pi/4\right)$$

$$\cos a \ \cos b - \sin a \ \sin b = \cos(b + a)$$



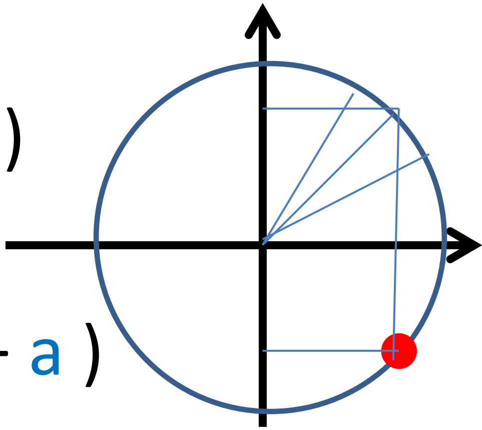
$$\frac{\sqrt{2}}{2}\cos x + \frac{-\sqrt{2}}{2}\sin x = \sin\left(x + 3\pi/4\right)$$

$$\sin a \ \cos b + \cos a \ \sin b = \sin(b + a)$$



$$\frac{-\sqrt{2}}{2}\cos x + \frac{\sqrt{2}}{2}\sin x = \sin\left(x + (-\pi/4)\right)$$

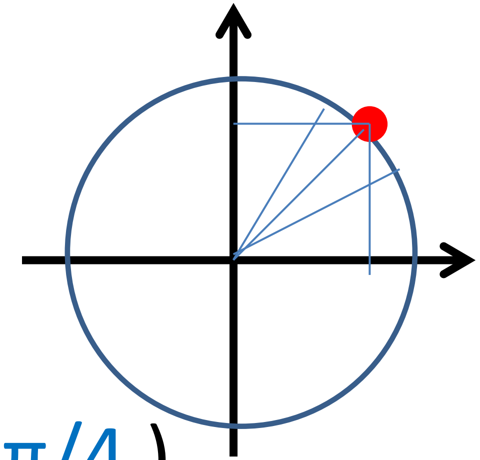
$$\sin(-\pi/4)\cos b + \cos(-\pi/4)\sin b = \sin(b + a)$$



... $(\cos x + \sin x) = \sin (x + \pi/4)$

$$\ldots\ (\cos x + \sin x) = \sin(x + \pi/4)$$

$$\sin \pi/4 \cos b + \cos \pi/4 \sin b = \sin(b + \pi/4)$$



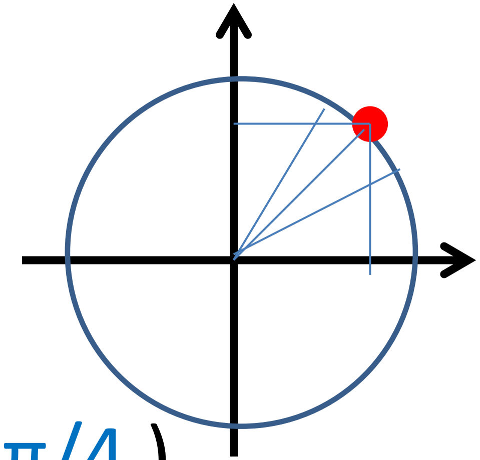
$\color{red}{\dots}$ (cos x + sin x) = sin (x + π/4)

$\sin \pi/4 \cos b + \cos \pi/4 \sin b = \sin(b + \pi/4)$

$$\frac{\sqrt{2}}{2} \cos x + \frac{\sqrt{2}}{2} \sin x = \sin (x + \pi/4)$$

$\color{red}{\dots}$ (cos x + sin x) = sin (x + π/4)

$$\ldots\ (\cos x + \sin x) = \sin(x + \pi/4)$$



$$\sin \pi/4 \cos b + \cos \pi/4 \sin b = \sin(b + \pi/4)$$

$$\frac{\sqrt{2}}{2}\cos x + \frac{\sqrt{2}}{2}\sin x = \sin(x + \pi/4)$$

$$\frac{\sqrt{2}}{2}(\cos x + \sin x) = \sin(x + \pi/4)$$

$3 \cos x - 3 \sin x = \ldots \cos (x + \pi/4)$

$$3 \cos x - 3 \sin x = \ldots \cos(x + \pi/4)$$

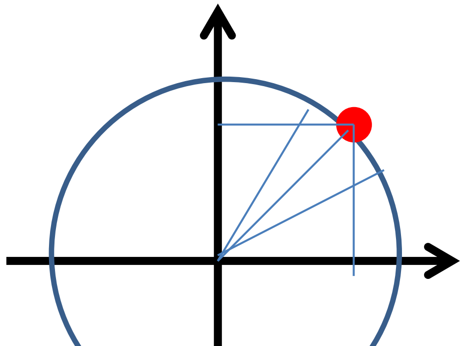
$$\cos(b + \pi/4) = \cos b \cos \pi/4 - \sin \pi/4 \sin b$$

$$\cos(x + \pi/4) = \ldots$$

$$3 \cos x - 3 \sin x = \ldots \cos (x + \pi/4)$$

$$\cos (b + \pi/4) = \cos b \cos \pi/4 - \sin \pi/4 \sin b$$

$$\cos (x + \pi/4) = \cos x \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \sin x$$

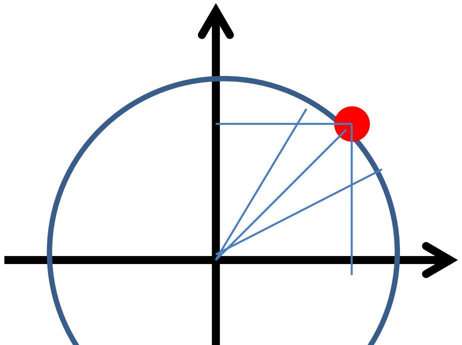


$$3\cos x - 3\sin x = \ldots \cos(x + \pi/4)$$

$$\cos(b + \pi/4) = \cos b \cos \pi/4 - \sin \pi/4 \sin b$$

$$\cos(x + \pi/4) = \cos x \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \sin x$$

$$\ldots \cos(x + \pi/4) = \ldots \left(\cos x \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \sin x \right)$$



$$= 3\cos x - 3\sin x$$

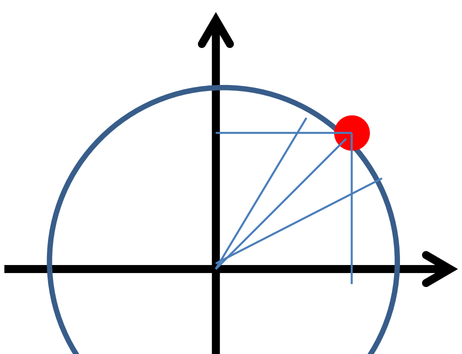
$$3 \cos x - 3 \sin x = \ldots \cos (x + \pi/4)$$

$$\cos (b + \pi/4) = \cos b \cos \pi/4 - \sin \pi/4 \sin b$$

$$\cos (x + \pi/4) = \cos x \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \sin x$$

$\ldots = 3 / (\sqrt{2} /2) = 3 \times (2/\sqrt{2}) = 3 \times \sqrt{2}$

$$\ldots \cos (x + \pi/4) = \ldots \left(\cos x \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \sin x \right)$$



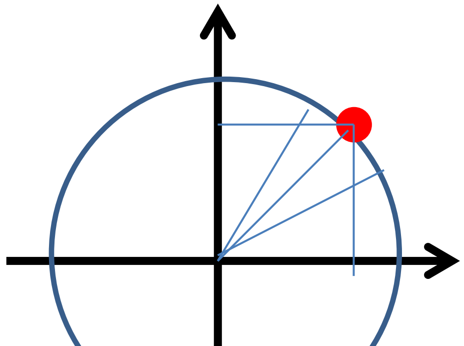
$$= 3 \cos x - 3 \sin x$$

$$3\cos x - 3\sin x = \ldots \cos(x + \pi/4)$$

$$\cos(b + \pi/4) = \cos b \cos \pi/4 - \sin \pi/4 \sin b$$

$$\cos(x + \pi/4) = \cos x \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \sin x$$

$$3\sqrt{2}\cos(x + \pi/4) = 3\sqrt{2}\left(\cos x \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \sin x\right)$$



$$= 3\cos x - 3\sin x$$

$\sqrt{3} \cos x + \sin x = \ldots \sin (x + \ldots)$

$\sqrt{3} \cos x + \sin x = \ldots \sin (x + \ldots)$

$\sqrt{3} \cos x + 1 \sin x$ *ne peut être* $\cos(b + a)$

ni $\sin(b + a)$

car …

$\sqrt{3} \cos x + \sin x = \ldots \sin (x + \ldots)$

$\sqrt{3} \cos x + 1 \sin x$ *ne peut être* $\cos(b + a)$

ni $\sin(b + a)$

car $\sqrt{3} > 1 \Rightarrow \sqrt{3}$ *ne peut être* $\cos a$ *ni* $\sin a$

$$\sqrt{3} \cos x + \sin x = \ldots \left(\ldots \cos x + \ldots \sin x \right)$$

$\sqrt{3} \cos x + \sin x = \ldots \sin (x + \ldots)$

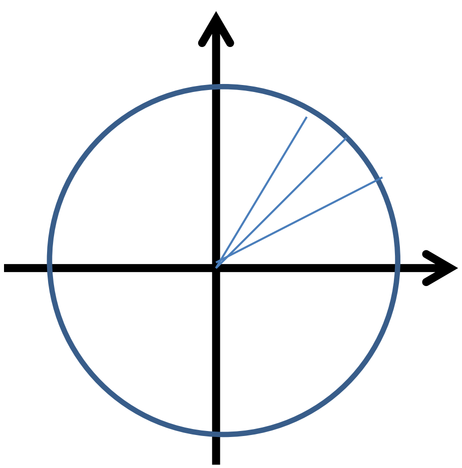
$\sqrt{3} \cos x + 1 \sin x$ *ne peut être* $\cos(b + a)$

ni $\sin(b + a)$

car $\sqrt{3} > 1 \Rightarrow \sqrt{3}$ *ne peut être* $\cos a$ *ni* $\sin a$

$$\sqrt{3} \cos x + \sin x = 2 \left(\frac{\sqrt{3}}{2} \cos x + \frac{1}{2} \sin x \right)$$

$$= 2 (\sin \ldots \cos x + \cos \ldots \sin x)$$



$\sqrt{3}\cos x + \sin x = \ldots \sin(x + \ldots)$

$\sqrt{3}\cos x + 1 \sin x$ *ne peut être* $\cos(b + a)$

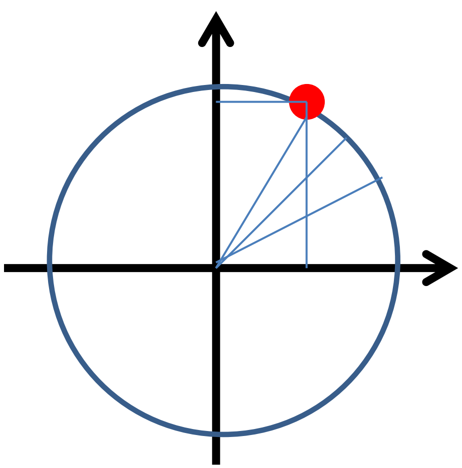
ni $\sin(b + a)$

car $\sqrt{3} > 1 \Rightarrow \sqrt{3}$ *ne peut être* $\cos a$ *ni* $\sin a$

$$\sqrt{3}\cos x + \sin x = 2\left(\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x\right)$$

$$= 2\left(\sin \pi/3 \cos x + \cos \pi/3 \sin x\right)$$

$$= \ldots \sin(x + \ldots)$$



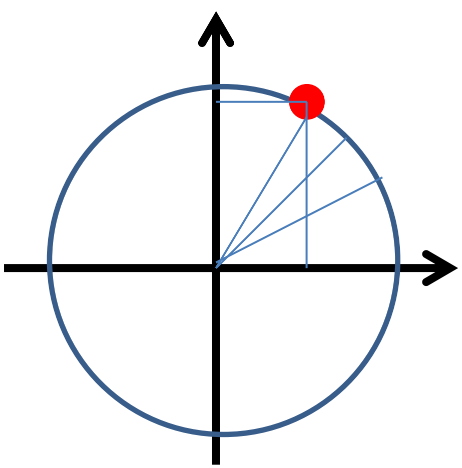
$\sqrt{3}\cos x + \sin x = \ldots \sin(x + \ldots)$

$\sqrt{3}\cos x + 1\sin x$ *ne peut être* $\cos(b + a)$

ni $\sin(b + a)$

car $\sqrt{3} > 1 \Rightarrow \sqrt{3}$ *ne peut être* $\cos a$ *ni* $\sin a$

$$\sqrt{3}\cos x + \sin x = 2\left(\frac{\sqrt{3}}{2}\cos x + \frac{1}{2}\sin x\right)$$



$$= 2(\sin \pi/3 \cos x + \cos \pi/3 \sin x)$$

$$= 2\sin(x + \pi/3)$$

$\cos x + \sqrt{3} \sin x = \ldots \sin (x - \ldots)$

$\cos x + \sqrt{3} \sin x = \ldots \sin (x - \ldots)$

$1 \cos x + \sqrt{3} \sin x$ *ne peut être* $\cos(b + a)$

ni $\sin(b + a)$

car $\sqrt{3} > 1$ $\Longrightarrow$ $\sqrt{3}$ *ne peut être* $\cos a$ *ni* $\sin a$

$$\cos x + \sqrt{3} \sin x = \ldots \left(\ldots \cos x + \ldots \sin x \right)$$

$$\cos x + \sqrt{3}\sin x = \ldots \sin(x - \ldots)$$

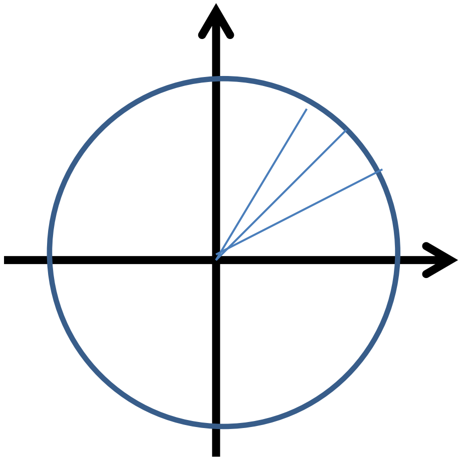
$1 \cos x + \sqrt{3} \sin x$ *ne peut être* $\cos(b + a)$

ni $\sin(b + a)$

car $\sqrt{3} > 1 \Longrightarrow \sqrt{3}$ *ne peut être* $\cos a$ *ni* $\sin a$

$$\cos x + \sqrt{3}\sin x = 2\left(\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x\right)$$

$$= 2(\sin \ldots \cos x + \cos \ldots \sin x)$$



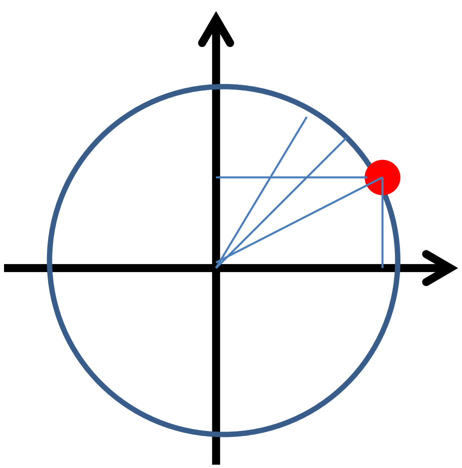
$\cos x + \sqrt{3} \sin x = \dots \sin (x - \dots)$

$1 \cos x + \sqrt{3} \sin x$ *ne peut être* $\cos(b + a)$

ni $\sin(b + a)$

car $\sqrt{3} > 1$ ⇒ $\sqrt{3}$ *ne peut être* $\cos a$ *ni* $\sin a$

$$\cos x + \sqrt{3} \sin x = 2 \left(\frac{1}{2} \cos x + \frac{\sqrt{3}}{2} \sin x \right)$$



$$= 2 (\sin \pi/6 \cos x + \cos \pi/6 \sin x)$$

$$= \dots \sin (x - \dots)$$

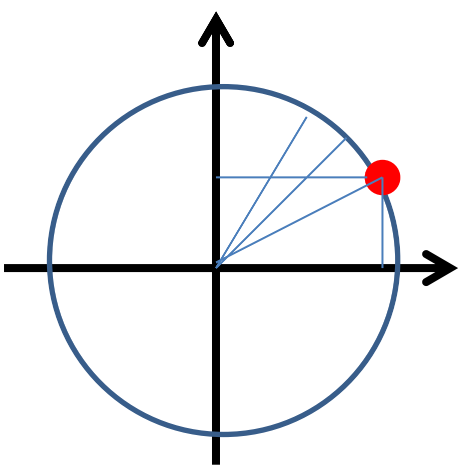
$\cos x + \sqrt{3} \sin x = \ldots \sin (x - \ldots)$

$1 \cos x + \sqrt{3} \sin x$ *ne peut être* $\cos(b + a)$

ni $\sin(b + a)$

car $\sqrt{3} > 1$ ⇒ $\sqrt{3}$ *ne peut être* $\cos a$ *ni* $\sin a$

$$\cos x + \sqrt{3} \sin x = 2 \left(\frac{1}{2} \cos x + \frac{\sqrt{3}}{2} \sin x \right)$$



$$= 2 (\sin \pi/6 \cos x + \cos \pi/6 \sin x)$$

$$= 2 \sin (x + \pi/6)$$

$$= 2 \sin (x - (- \pi/6))$$

$\cos x - \sin x = \ldots \cos (x - \ldots)$

$$\cos x - \sin x = \ldots \cos(x - \ldots)$$

$1 \cos x - 1 \sin x$ *ne peut être* $\cos(b + a)$

ni $\sin(b + a)$

car $\cos a = 1$ *et* $\sin a = 1 \Longrightarrow \cos^2 a + \sin^2 a \neq 1$

$$\cos x - \sin x = \ldots \left(\ldots \cos x - \ldots \sin x \right)$$

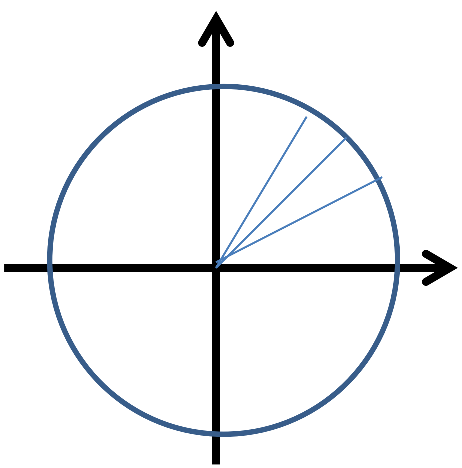
$\cos x - \sin x = \ldots \cos(x - \ldots)$

$1 \cos x - 1 \sin x$ *ne peut être* $\cos(b + a)$

ni $\sin(b + a)$

car $\cos a = 1$ *et* $\sin a = 1$ $\Rightarrow$ $\cos^2 a + \sin^2 a \neq 1$

$$\cos x - \sin x = \sqrt{2}\left(\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x\right)$$



$$= \sqrt{2}(\cos \ldots \cos x - \sin \ldots \sin x)$$

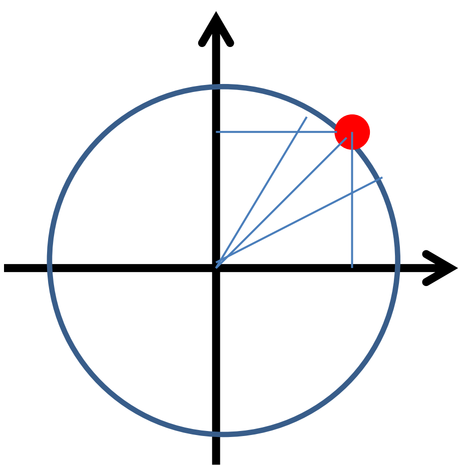
$\cos x - \sin x = \ldots \cos (x - \ldots)$

$1 \cos x - 1 \sin x$ *ne peut être* $\cos(b + a)$

ni $\sin(b + a)$

car $\cos a = 1$ *et* $\sin a = 1 \Rightarrow \cos^2 a + \sin^2 a \neq 1$

$$\cos x - \sin x = \sqrt{2} \left(\frac{\sqrt{2}}{2} \cos x - \frac{\sqrt{2}}{2} \sin x \right)$$



$$= \sqrt{2} (\cos \pi/4 \cos x - \sin \pi/4 \sin x)$$

$$= \ldots \cos (x - \ldots)$$

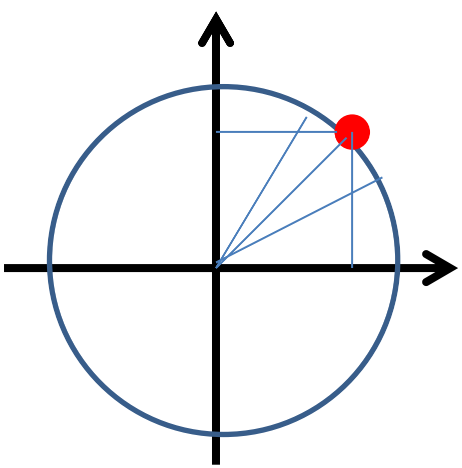
$\cos x - \sin x = \ldots \cos(x - \ldots)$

$1 \cos x - 1 \sin x$ *ne peut être* $\cos(b + a)$

ni $\sin(b + a)$

car $\cos a = 1$ *et* $\sin a = 1$ $\Rightarrow$ $\cos^2 a + \sin^2 a \neq 1$

$$\cos x - \sin x = \sqrt{2}\left(\frac{\sqrt{2}}{2}\cos x - \frac{\sqrt{2}}{2}\sin x\right)$$



$$= \sqrt{2}\,(\cos \pi/4 \cos x - \sin \pi/4 \sin x)$$

$$= \sqrt{2}\cos(x + \pi/4)$$

$$= \sqrt{2}\cos(x - (-\pi/4))$$

3°) Déterminez les réels A et B solutions de l'équation

$$\cos 4x - \sqrt{3} \sin 4x = A \sin (4x + B)$$

$\cos 4x - \sqrt{3} \sin 4x = A \sin(4x + B)$

$1 \cos 4x - \sqrt{3} \sin 4x$ *ne peut être* $\cos(b + a)$

ni $\sin(b + a)$

car $\sqrt{3} > 1$ $\Longrightarrow$ $\sqrt{3}$ *ne peut être* $\cos a$ *ni* $\sin a$

$$\cos 4x - \sqrt{3} \sin 4x = \ldots \left(\ldots \cos 4x - \ldots \sin 4x \right)$$

$\cos 4x \; - \; \sqrt{3} \sin 4x \; = A \sin (4x + B)$

$1 \cos 4x - \sqrt{3} \sin 4x$ *ne peut être* $\cos(b + a)$

ni $\sin(b + a)$

car $\sqrt{3} > 1 \Longrightarrow \sqrt{3}$ *ne peut être* cos a *ni* sin a

$$\cos 4x - \sqrt{3} \sin 4x = 2 \left(\frac{1}{2} \cos 4x - \frac{\sqrt{3}}{2} \sin 4x \right)$$

$$= A \sin (4x + B)$$

$$= A (\sin \ldots \cos 4x + \cos \ldots \sin 4x)$$

$\cos 4x - \sqrt{3} \sin 4x = A \sin(4x + B)$

$1 \cos 4x - \sqrt{3} \sin 4x$ *ne peut être* $\cos(b + a)$

ni $\sin(b + a)$

car $\sqrt{3} > 1 \Longrightarrow \sqrt{3}$ *ne peut être* $\cos a$ *ni* $\sin a$

$$\cos 4x - \sqrt{3} \sin 4x = 2\left(\frac{1}{2} \cos 4x + \frac{-\sqrt{3}}{2} \sin 4x\right)$$

$$= A \sin(4x + B)$$

$$= A(\sin \ldots \cos 4x + \cos \ldots \sin 4x)$$

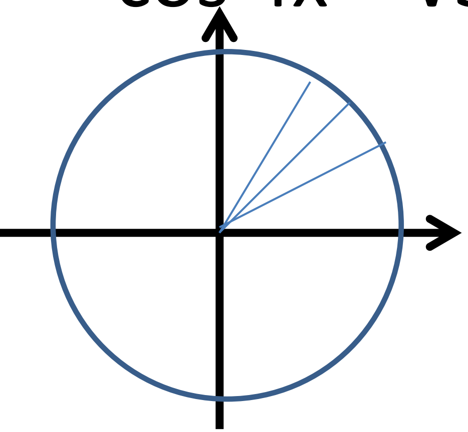
$\cos 4x - \sqrt{3} \sin 4x = A \sin (4x + B)$

$1 \cos 4x - \sqrt{3} \sin 4x$ *ne peut être* $\cos(b + a)$

ni $\sin(b + a)$

car $\sqrt{3} > 1$ $\Rightarrow$ $\sqrt{3}$ *ne peut être* $\cos a$ *ni* $\sin a$

$$\cos 4x - \sqrt{3} \sin 4x = 2 \left(\frac{1}{2} \cos 4x + \frac{-\sqrt{3}}{2} \sin 4x \right)$$



$$= A \sin (4x + B)$$

$$= A (\sin \ldots \cos 4x + \cos \ldots \sin 4x)$$

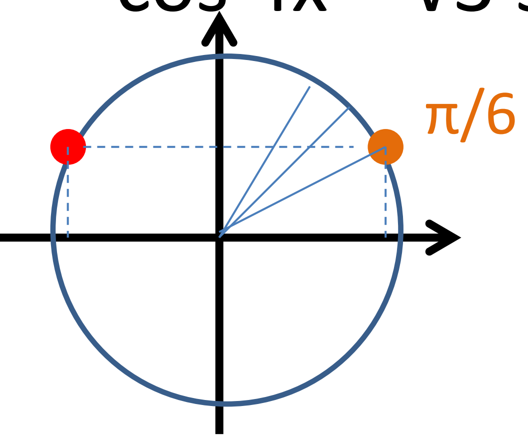
$\cos 4x - \sqrt{3} \sin 4x = A \sin (4x + B)$

$1 \cos 4x - \sqrt{3} \sin 4x$ *ne peut être* $\cos(b + a)$

ni $\sin(b + a)$

car $\sqrt{3} > 1 \Longrightarrow \sqrt{3}$ *ne peut être* $\cos a$ *ni* $\sin a$

$$\cos 4x - \sqrt{3} \sin 4x = 2 \left(\frac{1}{2} \cos 4x + \frac{-\sqrt{3}}{2} \sin 4x \right)$$



$$= 2 (\sin 5\pi/6 \cos 4x + \cos 5\pi/6 \sin 4x)$$

$$= 2 \sin (4x + 5\pi/6)$$

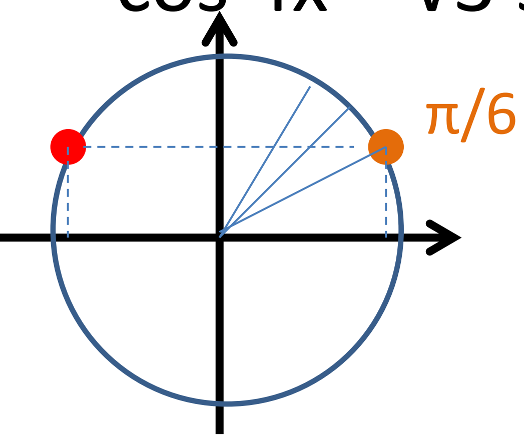
$\cos 4x \ - \ \sqrt{3}\sin 4x \ = A \sin(4x + B)$

$1 \cos 4x - \sqrt{3} \sin 4x$ *ne peut être* $\cos(b + a)$

ni $\sin(b + a)$

car $\sqrt{3} > 1 \Longrightarrow \sqrt{3}$ *ne peut être* $\cos a$ *ni* $\sin a$

$$\cos 4x - \sqrt{3}\sin 4x = 2\left(\frac{1}{2}\cos 4x + \frac{-\sqrt{3}}{2}\sin 4x\right)$$



$$= 2(\sin 5\pi/6 \cos 4x + \cos 5\pi/6 \sin 4x)$$

$$= 2 \sin(4x + 5\pi/6)$$

Réponses : combien de solutions $A = \ldots$?

combien de solutions $B = \ldots$?

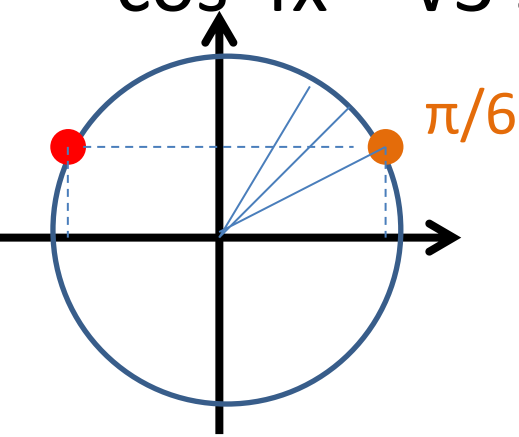
$\cos 4x - \sqrt{3} \sin 4x = A \sin(4x + B)$

$1 \cos 4x - \sqrt{3} \sin 4x$ *ne peut être* $\cos(b + a)$

ni $\sin(b + a)$

car $\sqrt{3} > 1 \Rightarrow \sqrt{3}$ *ne peut être* $\cos a$ *ni* $\sin a$

$$\cos 4x - \sqrt{3} \sin 4x = 2 \left(\frac{1}{2} \cos 4x + \frac{-\sqrt{3}}{2} \sin 4x \right)$$



$$= 2 \left(\sin 5\pi/6 \cos 4x + \cos 5\pi/6 \sin 4x \right)$$

$$= 2 \sin(4x + 5\pi/6)$$

Réponses : une *unique* solution $A = 2$

une *infinité* de solutions $B = 5\pi/6 + k2\pi$

$$\cos 4x - \sqrt{3} \sin 4x = A \sin(4x + B)$$

$$\cos 4x - \sqrt{3} \sin 4x = 2 \left(\frac{1}{2} \cos 4x + \frac{-\sqrt{3}}{2} \sin 4x \right)$$

$$= 2 \left(\sin 5\pi/6 \cos 4x + \cos 5\pi/6 \sin 4x \right)$$

$$= 2 \sin(4x + 5\pi/6)$$

Réponses : une *unique* solution $A = 2$

une *infinité* de solutions $B = 5\pi/6 + k2\pi$

Aucune erreur ? Aucune solution oubliée ?

$$\cos 4x - \sqrt{3} \sin 4x = A \sin(4x + B)$$

$$\cos 4x - \sqrt{3} \sin 4x = 2 \left(\frac{1}{2} \cos 4x + \frac{-\sqrt{3}}{2} \sin 4x \right)$$

$$= 2 \left(\sin 5\pi/6 \cos 4x + \cos 5\pi/6 \sin 4x \right)$$

$$= 2 \sin(4x + 5\pi/6)$$

Réponses : une *unique* solution $A = 2$

une *infinité* de solutions $B = 5\pi/6 + k2\pi$

Aucune erreur ?

Non !

Aucune solution oubliée ?

Peut-être ! Quelle étape peut donner plusieurs étapes ?

$$\cos 4x - \sqrt{3}\sin 4x = A \sin(4x + B)$$

$$\cos 4x - \sqrt{3}\sin 4x = 2\left(\frac{1}{2}\cos 4x + \frac{-\sqrt{3}}{2}\sin 4x\right)$$

Il y a plusieurs nombres ayant un même produit !

Il y a plusieurs nombres ayant une même somme !

Réponses pour **cette** possibilité : *unique* solution $A = 2$

et *infinité* de solutions $B = 5\pi/6 + k2\pi$

Aucune erreur ? Aucune solution oubliée ?

Non ! Peut-être ! Quelle étape peut donner plusieurs étapes ?

$$\cos 4x - \sqrt{3} \sin 4x = A \sin (4x + B)$$

$$= A (\ldots \qquad\qquad)$$

$$\cos 4x - \sqrt{3} \sin 4x = A \sin (4x + B)$$

$$= A (\sin 4x \cos B + \cos 4x \sin B)$$

$$= A \ldots \ + A \ldots$$

$$\cos 4x - \sqrt{3} \sin 4x = A \sin(4x + B)$$

$$= A(\sin 4x \cos B + \cos 4x \sin B)$$

$$= A \sin 4x \cos B + A \cos 4x \sin B$$

$$\Leftrightarrow \quad \ldots \cos 4x - \ldots \sin 4x = 0$$

$$\cos 4x - \sqrt{3} \sin 4x = A \sin (4x + B)$$

$$= A (\sin 4x \cos B + \cos 4x \sin B)$$

$$= A \sin 4x \cos B + A \cos 4x \sin B$$

$$\Leftrightarrow \cos 4x - A \cos 4x \sin B$$

$$- \sqrt{3} \sin 4x - A \sin 4x \cos B = 0$$

$$\Leftrightarrow (1 - A \sin B) \cos 4x$$

$$- (\sqrt{3} + A \cos B) \sin 4x = 0$$

On doit déterminer A et B

pour ... les valeurs de x

$$\cos 4x - \sqrt{3} \sin 4x = A \sin(4x + B)$$

$$= A (\sin 4x \cos B + \cos 4x \sin B)$$

$$= A \sin 4x \cos B + A \cos 4x \sin B$$

$$\Leftrightarrow \quad \cos 4x - A \cos 4x \sin B$$

$$- \sqrt{3} \sin 4x - A \sin 4x \cos B = 0$$

$$\Leftrightarrow \quad (1 - A \sin B) \cos 4x$$

$$- (\sqrt{3} + A \cos B) \sin 4x = 0$$

On doit déterminer A et B

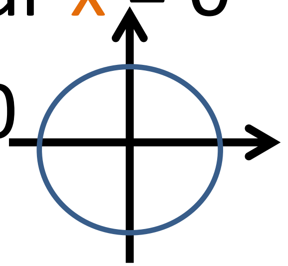
pour *toutes* les valeurs de x

$\cos 4x - \sqrt{3} \sin 4x = A \sin (4x + B)$

$\Leftrightarrow (1 - A \sin B) \cos 4x - (\sqrt{3} + A \cos B) \sin 4x = 0$

Cette égalité est vraie pour tout x donc pour x = 0

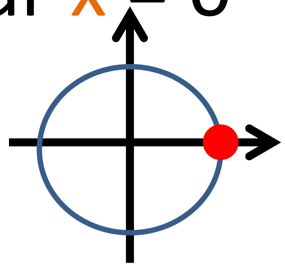
$\Rightarrow (1 - A \sin B) \times \ldots - (\sqrt{3} + A \cos B) \times \ldots = 0$



$$\cos 4x - \sqrt{3} \sin 4x = A \sin(4x + B)$$

$$\Leftrightarrow (1 - A \sin B) \cos 4x - (\sqrt{3} + A \cos B) \sin 4x = 0$$

Cette égalité est vraie pour tout x donc pour x = 0



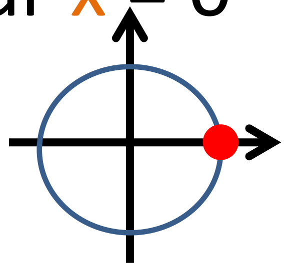
$$\Rightarrow (1 - A \sin B) \times 1 - (\sqrt{3} + A \cos B) \times 0 = 0$$

$$\Rightarrow 1 - A \sin B = 0$$

$\cos 4x - \sqrt{3} \sin 4x = A \sin(4x + B)$

$\Leftrightarrow (1 - A \sin B) \cos 4x - (\sqrt{3} + A \cos B) \sin 4x = 0$

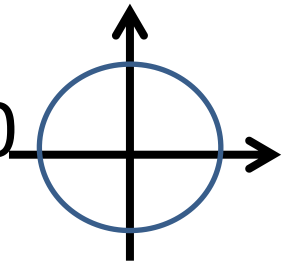
Cette égalité est vraie pour tout x donc pour $x = 0$



$\Rightarrow (1 - A \sin B) \times 1 - (\sqrt{3} + A \cos B) \times 0 = 0$

$\Rightarrow 1 - A \sin B = 0$

et aussi pour $x = \pi/8$

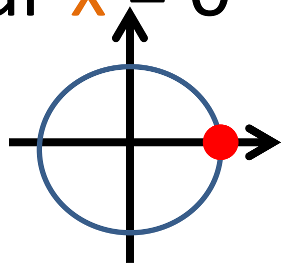


$\Rightarrow (1 - A \sin B) \times \ldots - (\sqrt{3} + A \cos B) \times \ldots = 0$

$\cos 4x - \sqrt{3} \sin 4x = A \sin (4x + B)$

$\Leftrightarrow (1 - A \sin B) \cos 4x - (\sqrt{3} + A \cos B) \sin 4x = 0$

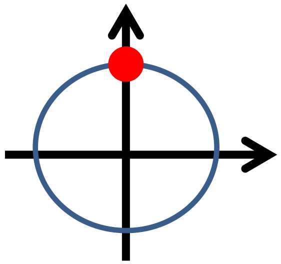
Cette égalité est vraie pour tout x donc pour $x = 0$



$\Rightarrow (1 - A \sin B) \times 1 - (\sqrt{3} + A \cos B) \times 0 = 0$

$\Rightarrow 1 - A \sin B = 0$

et aussi pour $x = \pi/8$



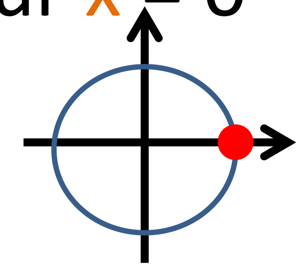
$\Rightarrow (1 - A \sin B) \times 0 - (\sqrt{3} + A \cos B) \times 1 = 0$

$\Rightarrow \sqrt{3} + A \cos B = 0$

$\cos 4x - \sqrt{3} \sin 4x = A \sin (4x + B)$

$\Longleftrightarrow (1 - A \sin B) \cos 4x - (\sqrt{3} + A \cos B) \sin 4x = 0$

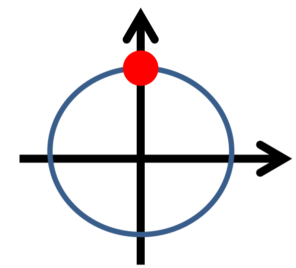
Cette égalité est vraie pour tout x donc pour $x = 0$



$\Longrightarrow (1 - A \sin B) \times 1 - (\sqrt{3} + A \cos B) \times 0 = 0$

$\Longrightarrow 1 - A \sin B = 0$

et aussi pour $x = \pi/8$



$\Longrightarrow (1 - A \sin B) \times 0 - (\sqrt{3} + A \cos B) \times 1 = 0$

Méthode générale :

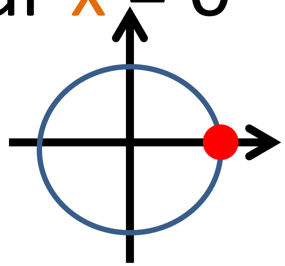
$\Longrightarrow \sqrt{3} + A \cos B = 0$

Cette égalité est vraie pour tout x donc la seule possibilité est ... $\cos 4x$ – ... $\sin 4x = 0$

$\cos 4x - \sqrt{3} \sin 4x = A \sin(4x + B)$

$\Leftrightarrow (1 - A \sin B) \cos 4x - (\sqrt{3} + A \cos B) \sin 4x = 0$

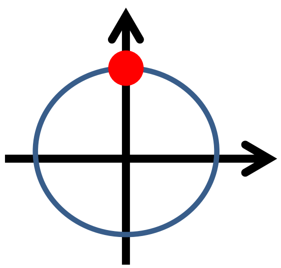
Cette égalité est vraie pour tout x donc pour x = 0



$\Rightarrow (1 - A \sin B) \times 1 - (\sqrt{3} + A \cos B) \times 0 = 0$

$\Rightarrow 1 - A \sin B = 0$

et aussi pour x = π/8



$\Rightarrow (1 - A \sin B) \times 0 - (\sqrt{3} + A \cos B) \times 1 = 0$

Méthode générale : $\Rightarrow \sqrt{3} + A \cos B = 0$

Cette égalité est vraie pour tout x donc la seule possibilité est $0 \cos 4x - 0 \sin 4x = 0$

$\Rightarrow 1 - A \sin B = 0$ et $\sqrt{3} + A \cos B = 0$

$\cos 4x - \sqrt{3} \sin 4x = A \sin (4x + B)$

$\Leftrightarrow (1 - A \sin B) \cos 4x - (\sqrt{3} + A \cos B) \sin 4x = 0$

Cette égalité est vraie pour tout x donc la seule possibilité est $0 \cos 4x - 0 \sin 4x = 0$

$\Rightarrow 1 - A \sin B = 0$ et $\sqrt{3} + A \cos B = 0$

$\Rightarrow \sin B = 1/A$ et $\cos B = - \sqrt{3}/A$

B existe seulement si c'est un réel du cercle trigo

$\Rightarrow$ …

$\cos 4x - \sqrt{3} \sin 4x = A \sin (4x + B)$

$\Leftrightarrow (1 - A \sin B) \cos 4x - (\sqrt{3} + A \cos B) \sin 4x = 0$

Cette égalité est vraie pour tout x donc la seule possibilité est $0 \cos 4x - 0 \sin 4x = 0$

$\Rightarrow 1 - A \sin B = 0$ et $\sqrt{3} + A \cos B = 0$

$\Rightarrow \sin B = 1/A$ et $\cos B = - \sqrt{3}/A$

B existe seulement si c'est un réel du cercle trigo

$\Rightarrow \cos^2 B + \sin^2 B = 1$

$$\Rightarrow \left(\frac{-\sqrt{3}}{A} \right)^2 + \left(\frac{1}{A} \right)^2 = 1 \Rightarrow A = \ldots$$

$\cos 4x - \sqrt{3} \sin 4x = A \sin(4x + B)$

$\Leftrightarrow (1 - A \sin B) \cos 4x - (\sqrt{3} + A \cos B) \sin 4x = 0$

Cette égalité est vraie pour tout x donc la seule possibilité est $0 \cos 4x - 0 \sin 4x = 0$

$\Rightarrow 1 - A \sin B = 0$ et $\sqrt{3} + A \cos B = 0$

$\Rightarrow \sin B = 1/A$ et $\cos B = -\sqrt{3}/A$

B existe seulement si c'est un réel du cercle trigo

$\Rightarrow \cos^2 B + \sin^2 B = 1$

$$\Rightarrow \left(\frac{-\sqrt{3}}{A}\right)^2 + \left(\frac{1}{A}\right)^2 = 1 \Rightarrow \frac{4}{A^2} = 1$$

$\cos 4x - \sqrt{3} \sin 4x = A \sin (4x + B)$

$\Leftrightarrow (1 - A \sin B) \cos 4x - (\sqrt{3} + A \cos B) \sin 4x = 0$

Cette égalité est vraie pour tout x donc la seule possibilité est $0 \cos 4x - 0 \sin 4x = 0$

$\Rightarrow 1 - A \sin B = 0$ et $\sqrt{3} + A \cos B = 0$

$\Rightarrow \sin B = 1/A$ et $\cos B = - \sqrt{3}/A$

B existe seulement si c'est un réel du cercle trigo

$\Rightarrow \cos^2 B + \sin^2 B = 1$

$$\Rightarrow \left(\frac{-\sqrt{3}}{A} \right)^2 + \left(\frac{1}{A} \right)^2 = 1 \Rightarrow \frac{4}{A^2} = 1 \Rightarrow A^2 = 4$$

$\cos 4x - \sqrt{3} \sin 4x = A \sin (4x + B)$

$\Longleftrightarrow (1 - A \sin B) \cos 4x - (\sqrt{3} + A \cos B) \sin 4x = 0$

Cette égalité est vraie pour tout x donc la seule possibilité est $0 \cos 4x - 0 \sin 4x = 0$

$\Rightarrow 1 - A \sin B = 0$ et $\sqrt{3} + A \cos B = 0$

$\Rightarrow \sin B = 1/A$ et $\cos B = - \sqrt{3}/A$

B existe seulement si c'est un réel du cercle trigo

$\Rightarrow \cos^2 B + \sin^2 B = 1$

$$\Rightarrow \left(\frac{-\sqrt{3}}{A} \right)^2 + \left(\frac{1}{A} \right)^2 = 1 \Rightarrow \frac{4}{A^2} = 1 \Rightarrow A = 2 \text{ ou } -2$$

$\cos 4x - \sqrt{3} \sin 4x = A \sin(4x + B)$ $\forall$: *quel que soit*

$\Leftrightarrow (1 - A \sin B) \cos 4x - (\sqrt{3} + A \cos B) \sin 4x = 0$

$\boxed{\text{vrai } \forall x \Rightarrow 1 - A \sin B = 0 \text{ et } \sqrt{3} + A \cos B = 0}$

$\Rightarrow \sin B = 1/A$ et $\cos B = -\sqrt{3}/A$

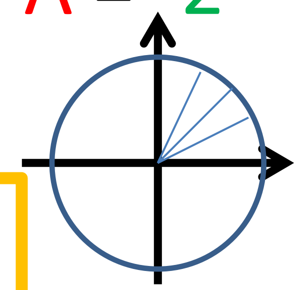
B existe $\Rightarrow \cos^2 B + \sin^2 B = 1$

$\Rightarrow (-\sqrt{3}/A)^2 + (1/A)^2 = 1 \Rightarrow 3/A^2 + 1/A^2 = 1$

$\Rightarrow 4/A^2 = 1 \Rightarrow A^2 = 4 \Rightarrow A = 2$ ou $A = -2$

$\boxed{A = 2}$

$\Rightarrow \boxed{B = \ldots}$



$\cos 4x - \sqrt{3} \sin 4x = A \sin(4x + B)$ $\forall$: *quel que soit*

$\Leftrightarrow (1 - A \sin B) \cos 4x - (\sqrt{3} + A \cos B) \sin 4x = 0$

$\boxed{\text{vrai } \forall x \Rightarrow 1 - A \sin B = 0 \text{ et } \sqrt{3} + A \cos B = 0}$

$\Rightarrow \sin B = 1/A$ et $\cos B = -\sqrt{3}/A$

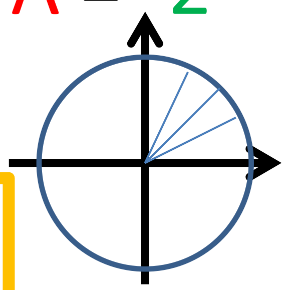
B existe $\Rightarrow \cos^2 B + \sin^2 B = 1$

$\Rightarrow (-\sqrt{3}/A)^2 + (1/A)^2 = 1 \Rightarrow 3/A^2 + 1/A^2 = 1$

$\Rightarrow 4/A^2 = 1 \Rightarrow A^2 = 4 \Rightarrow A = 2$ ou $A = -2$

$\boxed{A = 2} \Rightarrow \sin B = 1/2$ et $\cos B = -\sqrt{3}/2$

$\Rightarrow \boxed{B = \ldots}$



$\cos 4x - \sqrt{3} \sin 4x = A \sin(4x + B)$ $\qquad \forall$: *quel que soit*

$\Leftrightarrow (1 - A \sin B) \cos 4x - (\sqrt{3} + A \cos B) \sin 4x = 0$

$\boxed{\text{vrai } \forall x \Rightarrow 1 - A \sin B = 0 \quad \text{et} \quad \sqrt{3} + A \cos B = 0}$

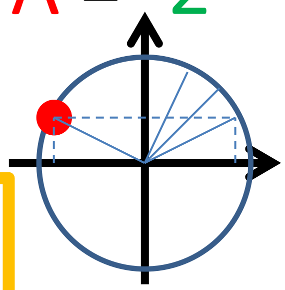
$\Rightarrow \sin B = 1/A \quad \text{et} \quad \cos B = -\sqrt{3}/A$

B existe $\Rightarrow \cos^2 B + \sin^2 B = 1$

$\Rightarrow (-\sqrt{3}/A)^2 + (1/A)^2 = 1 \Rightarrow 3/A^2 + 1/A^2 = 1$

$\Rightarrow 4/A^2 = 1 \Rightarrow A^2 = 4 \Rightarrow A = 2$ ou $A = -2$

$\boxed{A = 2} \Rightarrow \sin B = 1/2 \text{ et } \cos B = -\sqrt{3}/2$



Solutions déjà trouvées $\Rightarrow \boxed{B = 5\pi/6 + k2\pi}$

$\cos 4x - \sqrt{3} \sin 4x = A \sin(4x + B)$ $\forall$: *quel que soit*

$\Leftrightarrow (1 - A \sin B) \cos 4x - (\sqrt{3} + A \cos B) \sin 4x = 0$

$\boxed{\text{vrai } \forall x \Rightarrow 1 - A \sin B = 0 \text{ et } \sqrt{3} + A \cos B = 0}$

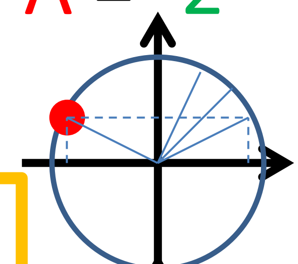
$\Rightarrow \sin B = 1/A$ et $\cos B = -\sqrt{3}/A$

B existe $\Rightarrow \cos^2 B + \sin^2 B = 1$

$\Rightarrow (-\sqrt{3}/A)^2 + (1/A)^2 = 1 \Rightarrow 3/A^2 + 1/A^2 = 1$

$\Rightarrow 4/A^2 = 1 \Rightarrow A^2 = 4 \Rightarrow A = 2$ ou $A = -2$

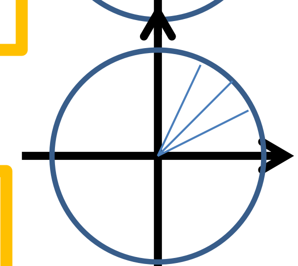
$\boxed{A = 2} \Rightarrow \sin B = 1/2$ et $\cos B = -\sqrt{3}/2$



Solutions déjà trouvées

$\Rightarrow \boxed{B = 5\pi/6 + k2\pi}$

$\boxed{A = -2}$



$\Rightarrow \boxed{B = \ldots}$

$\cos 4x - \sqrt{3} \sin 4x = A \sin(4x + B)$ $\forall$: *quel que soit*

$\Leftrightarrow (1 - A \sin B) \cos 4x - (\sqrt{3} + A \cos B) \sin 4x = 0$

$\boxed{\text{vrai } \forall x \Rightarrow 1 - A \sin B = 0 \quad \text{et} \quad \sqrt{3} + A \cos B = 0}$

$\Rightarrow \sin B = 1/A \quad \text{et} \quad \cos B = -\sqrt{3}/A$

B existe $\Rightarrow \cos^2 B + \sin^2 B = 1$

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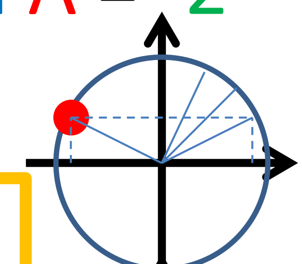
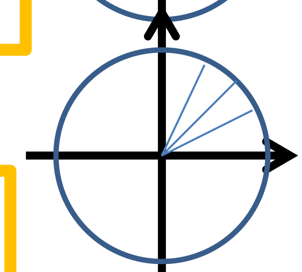
$\boxed{A = 2} \Rightarrow \sin B = 1/2 \quad \text{et} \quad \cos B = -\sqrt{3}/2$



Solutions déjà trouvées $\Rightarrow \boxed{B = 5\pi/6 + k2\pi}$

$\boxed{A = -2} \Rightarrow \sin B = -1/2 \quad \text{et} \quad \cos B = \sqrt{3}/2$



$\Rightarrow \boxed{B = \ldots}$

$\cos 4x - \sqrt{3} \sin 4x = A \sin(4x + B)$ $\forall$: *quel que soit*

$\Leftrightarrow (1 - A \sin B) \cos 4x - (\sqrt{3} + A \cos B) \sin 4x = 0$

$\boxed{\text{vrai } \forall x \Rightarrow 1 - A \sin B = 0 \quad \text{et} \quad \sqrt{3} + A \cos B = 0}$

$\Rightarrow \sin B = 1/A \quad \text{et} \quad \cos B = -\sqrt{3}/A$

B existe $\Rightarrow \cos^2 B + \sin^2 B = 1$

$\Rightarrow (-\sqrt{3}/A)^2 + (1/A)^2 = 1 \Rightarrow 3/A^2 + 1/A^2 = 1$

$\Rightarrow 4/A^2 = 1 \Rightarrow A^2 = 4 \Rightarrow A = 2$ ou $A = -2$

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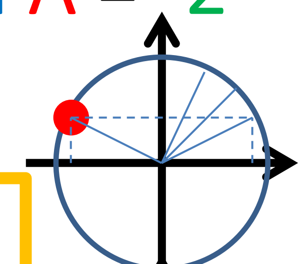
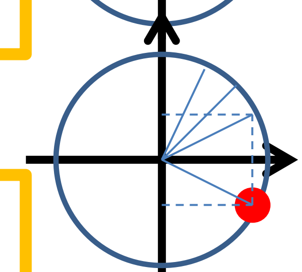


Solutions déjà trouvées $\Rightarrow \boxed{B = 5\pi/6 + k2\pi}$

$\boxed{A = -2} \Rightarrow \sin B = -1/2 \quad \text{et} \quad \cos B = \sqrt{3}/2$



Solutions nouvelles $\Rightarrow \boxed{B = -\pi/6 + k2\pi}$

Exercice 12 :

Déterminez les primitives F des fonctions f suivantes :

$5 (\sin x)^4 \cos x$

$2 (\cos x)^1 \sin x$

$\sin 2x$

$3 (\sin x)^2 (- \cos x)$

$\cos x (\sin x)^9$

$\sin x (\cos x)^3$

$(\cos x)^4$

$3 \cos^2 x$

$\sin^2 x$

$5\,(\sin x)^4 \cos x \Rightarrow \ldots$

$5(\sin x)^4 \cos x = 5u^4 \times u' = (u^5)' \Rightarrow (\sin x)^5 + C$

$2(\cos x)^1 \sin x$

$5\,(\sin x)^4 \cos x = 5u^4 \times u' = (u^5)' \Rightarrow (\sin x)^5 + C$

$2\,(\cos x)^1 \sin x = -\,2\,(\cos x)^1\,(-\sin x)$

$= -\,1\,(2u \times u') = -\,1\,(u^2)' \Rightarrow -\,(\cos x)^2 + C$

$\sin 2x$

$5\,(\sin x)^4 \cos x = 5u^4 \times u' = (u^5)' \Rightarrow (\sin x)^5 + C$

$2\,(\cos x)^1 \sin x = -\,2\,(\cos x)^1 (-\sin x)$

$= -\,1\,(2u \times u') = -\,1\,(u^2)' \Rightarrow -\,(\cos x)^2 + C$

$\sin 2x = \sin u = -\cos' u = -\,0{,}5 \cos' u \times u' = -\,0{,}5\,(\cos u)'$

$\Rightarrow -\,0{,}5 \cos 2x + C$

$5\ (\sin x)^4 \cos x = 5u^4 \times u' = (u^5)'$ $\Rightarrow$ $(\sin x)^5 + C$

$2\ (\cos x)^1 \sin x = -\ 2\ (\cos x)^1\ (-\sin x)$

$= -\ 1\ (2u \times u') = -\ 1\ (u^2)'$ $\Rightarrow$ $-\ (\cos x)^2 + C$

$\sin 2x = \sin u = -\cos' u = -\ 0{,}5 \cos' u \times u' = -\ 0{,}5\ (\cos u)'$

$\Rightarrow$ $-\ 0{,}5 \cos 2x + C$

Remarque :

comme $\sin 2x = 2\ (\cos x)^1 \sin x$

ce sont les mêmes fonctions

$\Rightarrow$ on doit trouver …

$5\,(\sin x)^4 \cos x = 5u^4 \times u' = (u^5)'$ ⇒ $(\sin x)^5 + C$

$2\,(\cos x)^1 \sin x = -\,2\,(\cos x)^1\,(-\sin x)$

$= -\,1\,(2u \times u') = -\,1\,(u^2)'$ ⇒ $-\,(\cos x)^2 + C$

$\sin 2x = \sin u = -\cos' u = -\,0{,}5 \cos' u \times u' = -\,0{,}5\,(\cos u)'$

⇒ $-\,0{,}5 \cos 2x + C$

Remarque :

comme $\sin 2x = 2\,(\cos x)^1 \sin x$

ce sont les mêmes fonctions

⇒ on doit trouver les mêmes primitives

$-\,(\cos x)^2 + C = -\,0{,}5 \cos 2x + C$?

$\cos^2 x - C = 0{,}5 \cos 2x - C$?

$\cos^2 x = 0{,}5 \cos 2x$? Mais …

$5\,(\sin x)^4 \cos x = 5u^4 \times u' = (u^5)'$ ⟹ $(\sin x)^5 + C$

$2\,(\cos x)^1 \sin x = -\,2\,(\cos x)^1\,(-\sin x)$

$= -\,1\,(2u \times u') = -\,1\,(u^2)'$ ⟹ $-\,(\cos x)^2 + C$

$\sin 2x = \sin u = -\cos' u = -\,0{,}5 \cos' u \times u' = -\,0{,}5\,(\cos u)'$

⟹ $-\,0{,}5 \cos 2x + C$

Remarque :

comme $\sin 2x = 2\,(\cos x)^1 \sin x$

ce sont les mêmes fonctions

⟹ on doit trouver les mêmes primitives

$-\,(\cos x)^2 + C = -\,0{,}5 \cos 2x + C$?

$\cos^2 x - C = 0{,}5 \cos 2x - C$?

$\cos^2 x = 0{,}5 \cos 2x$? Mais les deux fonctions n'ont pas forcément la même constante d'intégration !

$5\,(\sin x)^4 \cos x = 5u^4 \times u' = (u^5)' \Rightarrow (\sin x)^5 + C$

$2\,(\cos x)^1 \sin x = -\,2\,(\cos x)^1\,(-\sin x)$

$= -\,1\,(2u \times u') = -\,1\,(u^2)' \Rightarrow -\,(\cos x)^2 + C$

$\sin 2x = \sin u = -\cos' u = -\,0{,}5 \cos' u \times u' = -\,0{,}5\,(\cos u)'$

$\Rightarrow -\,0{,}5 \cos 2x + C$

Remarque :

comme $\sin 2x = 2\,(\cos x)^1 \sin x$

ce sont les mêmes fonctions

$\Rightarrow$ on doit trouver les mêmes primitives

$-\,(\cos x)^2 + C = -\,0{,}5 \cos 2x + D$?

$\cos^2 x - C = 0{,}5 \cos 2x - D$?

$\cos^2 x = 0{,}5 \cos 2x + C - D$?

$5\,(\sin x)^4 \cos x = 5u^4 \times u' = (u^5)' \Rightarrow (\sin x)^5 + C$

$2\,(\cos x)^1 \sin x = -\,2\,(\cos x)^1\,(-\sin x)$

$= -\,1\,(2u \times u') = -\,1\,(u^2)' \Rightarrow -\,(\cos x)^2 + C$

$\sin 2x = \sin u = -\cos' u = -\,0{,}5 \cos' u \times u' = -\,0{,}5\,(\cos u)'$

$\Rightarrow -\,0{,}5 \cos 2x + C$

Remarque :

comme $\sin 2x = 2\,(\cos x)^1 \sin x$

ce sont les mêmes fonctions

$\Rightarrow$ on doit trouver les mêmes primitives

$-\,(\cos x)^2 + C = -\,0{,}5 \cos 2x + D$?

$\cos^2 x - C = 0{,}5 \cos 2x - D$?

$\cos^2 x = 0{,}5 \cos 2x + C - D$? Oui !

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$3 (\sin x)^2 (- \cos x)$

$3 (\sin x)^2 (- \cos x) = - 3 (\sin x)^2 \cos x$

$= - 3u^2 \times u' = (u^3)'$ $\Rightarrow$ $- (\sin x)^3 + C$

$\cos x (\sin x)^9$

$3 (\sin x)^2 (- \cos x) = - 3 (\sin x)^2 \cos x$

$= - 3u^2 \times u' = (u^3)' \Rightarrow - (\sin x)^3 + C$

$\cos x (\sin x)^9 = 0{,}1 \times 10 (\sin x)^9 \cos x$

$= 0{,}1 \times 10u^9 \times u' = 0{,}1 (u^{10})' \Rightarrow 0{,}1 (\sin x)^{10} + C$

$\sin x (\cos x)^3$

$$3\,(\sin x)^2\,(-\cos x) = -3\,(\sin x)^2 \cos x$$

$$= -3u^2 \times u' = (u^3)' \Rightarrow -(\sin x)^3 + C$$

$$\cos x\,(\sin x)^9 = 0{,}1 \times 10\,(\sin x)^9 \cos x$$

$$= 0{,}1 \times 10u^9 \times u' = 0{,}1\,(u^{10})' \Rightarrow 0{,}1\,(\sin x)^{10} + C$$

$$\sin x\,(\cos x)^3 = -0{,}25 \times 4\,(\cos x)^3\,(-\sin x)$$

$$= -0{,}25 \times 4u^3 \times u' = -0{,}25\,(u^4)'$$

$$\Rightarrow -0{,}25\,(\cos x)^4 + C$$

$$(\cos x)^4$$

$3 (\sin x)^2 (- \cos x) = - 3 (\sin x)^2 \cos x$

$= - 3u^2 \times u' = (u^3)'$ ⇨ $- (\sin x)^3 + C$

$\cos x (\sin x)^9 = 0,1 \times 10 (\sin x)^9 \cos x$

$= 0,1 \times 10u^9 \times u' = 0,1 (u^{10})'$ ⇨ $0,1 (\sin x)^{10} + C$

$\sin x (\cos x)^3 = - 0,25 \times 4 (\cos x)^3 (- \sin x)$

$= - 0,25 \times 4 u^3 \times u' = - 0,25 (u^4)'$

⇨ $- 0,25 (\cos x)^4 + C$

$(\cos x)^4 = 0,2 \times 5u^4$ *mais il manque le* $\times u'$ ⇨ impossible

$3 \cos^2 x$

$3 (\sin x)^2 (- \cos x) = - 3 (\sin x)^2 \cos x$

$= - 3u^2 \times u' = (u^3)' \Rightarrow - (\sin x)^3 + C$

$\cos x (\sin x)^9 = 0{,}1 \times 10 (\sin x)^9 \cos x$

$= 0{,}1 \times 10u^9 \times u' = 0{,}1 (u^{10})' \Rightarrow 0{,}1 (\sin x)^{10} + C$

$\sin x (\cos x)^3 = - 0{,}25 \times 4 (\cos x)^3 (- \sin x)$

$= - 0{,}25 \times 4 u^3 \times u' = - 0{,}25 (u^4)'$

$\Rightarrow - 0{,}25 (\cos x)^4 + C$

$(\cos x)^4 = 0{,}2 \times 5u^4$ *mais il manque le* $\times u'$ $\Rightarrow$ impossible

$3 \cos^2 x = 3u^2$ *mais il manque le* $\times u'$ $\Rightarrow$ impossible

$$3\,(\sin x)^2\,(-\cos x) = -3\,(\sin x)^2 \cos x$$

$$= -3u^2 \times u' = (u^3)' \Rightarrow -(\sin x)^3 + C$$

$$\cos x\,(\sin x)^9 = 0{,}1 \times 10\,(\sin x)^9 \cos x$$

$$= 0{,}1 \times 10u^9 \times u' = 0{,}1\,(u^{10})' \Rightarrow 0{,}1\,(\sin x)^{10} + C$$

$$\sin x\,(\cos x)^3 = -0{,}25 \times 4\,(\cos x)^3\,(-\sin x)$$

$$= -0{,}25 \times 4\,u^3 \times u' = -0{,}25\,(u^4)'$$

$$\Rightarrow -0{,}25\,(\cos x)^4 + C$$

$(\cos x)^4 = 0{,}2 \times 5u^4$ *mais il manque le* $\times\, u'$ $\Rightarrow$ impossible

$3\cos^2 x = 3u^2$ *mais il manque le* $\times\, u'$ $\Rightarrow$ impossible

$$3\cos^2 x = 3\,(0{,}5\cos 2x + 0{,}5)$$

$$\cos^2 x = \frac{\cos 2x + 1}{2}$$ *formule de linéarisation*

$3\,(\sin x)^2\,(-\cos x) = -3\,(\sin x)^2 \cos x$

$= -3u^2 \times u' = (u^3)'$ ⇒ $-(\sin x)^3 + C$

$\cos x\,(\sin x)^9 = 0{,}1 \times 10\,(\sin x)^9 \cos x$

$= 0{,}1 \times 10u^9 \times u' = 0{,}1\,(u^{10})'$ ⇒ $0{,}1\,(\sin x)^{10} + C$

$\sin x\,(\cos x)^3 = -0{,}25 \times 4\,(\cos x)^3\,(-\sin x)$

$= -0{,}25 \times 4\,u^3 \times u' = -0{,}25\,(u^4)'$

⇒ $-0{,}25\,(\cos x)^4 + C$

$(\cos x)^4 = 0{,}2 \times 5u^4$ *mais il manque le* $\times\, u'$ ⇒ impossible

$3\cos^2 x = 3u^2$ *mais il manque le* $\times\, u'$ ⇒ impossible mais…

$3\cos^2 x = 3\,(0{,}5\cos 2x + 0{,}5) = 1{,}5\cos 2x + 1{,}5$

⇒ $1{,}5 \times 0{,}5\sin 2x + 1{,}5x + C$

$$\cos^2 x = \frac{\cos 2x + 1}{2}$$

formule de linéarisation

$(\cos x)^4 = 0{,}2 \times 5u^4$ *mais il manque le* $\times u'$ ⇨ impossible

$3 \cos^2 x = 3u^2$ *mais il manque le* $\times u'$ ⇨ impossible mais…

$3 \cos^2 x = 3 (0{,}5 \cos 2x + 0{,}5) = 1{,}5 \cos 2x + 1{,}5$

$$\cos^2 x = \frac{\cos 2x + 1}{2}$$ *formule de linéarisation*

⇨ $1{,}5 \times 0{,}5 \sin 2x + 1{,}5x + C$

$\sin^2 x$

$(\cos x)^4 = 0{,}2 \times 5u^4$ *mais il manque le* $\times\ u'$ ➡ impossible

$3 \cos^2 x = 3u^2$ *mais il manque le* $\times\ u'$ ➡ impossible mais…

$3 \cos^2 x = 3\,(0{,}5 \cos 2x + 0{,}5) = 1{,}5 \cos 2x + 1{,}5$

$$\cos^2 x = \frac{\cos 2x + 1}{2}$$ *formule de linéarisation*

➡ $1{,}5 \times 0{,}5 \sin 2x + 1{,}5x + C$

$\sin^2 x = 1/3 \times 3u^2$ *mais il manque le* $\times\ u'$ ➡ impossible mais…

$(\cos x)^4 = 0{,}2 \times 5u^4$ *mais il manque le* $\times u'$ ⇨ impossible

$3\cos^2 x = 3u^2$ *mais il manque le* $\times u'$ ⇨ impossible mais…

$3\cos^2 x = 3\,(0{,}5\cos 2x + 0{,}5) = 1{,}5\cos 2x + 1{,}5$

$$\cos^2 x = \frac{\cos 2x + 1}{2}$$ *formule de linéarisation*

⇨ $1{,}5 \times 0{,}5\sin 2x + 1{,}5x + C$

$\sin^2 x = 1/3 \times 3u^2$ *mais il manque le* $\times u'$ ⇨ impossible mais…

$\sin^2 x = 0{,}5 - 0{,}5\cos 2x$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$ *formule de linéarisation*

⇨ …

$(\cos x)^4 = 0{,}2 \times 5u^4$ *mais il manque le* $\times u'$ ⇨ impossible

$3\cos^2 x = 3u^2$ *mais il manque le* $\times u'$ ⇨ impossible mais…

$3\cos^2 x = 3(0{,}5\cos 2x + 0{,}5) = 1{,}5\cos 2x + 1{,}5$

$$\cos^2 x = \frac{\cos 2x + 1}{2}$$ *formule de linéarisation*

⇨ $1{,}5 \times 0{,}5\sin 2x + 1{,}5x + C$

$\sin^2 x = 1/3 \times 3u^2$ *mais il manque le* $\times u'$ ⇨ impossible mais…

$\sin^2 x = 0{,}5 - 0{,}5\cos 2x$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$ *formule de linéarisation*

⇨ $0{,}5x - 0{,}5 \times 0{,}5\sin 2x + C$